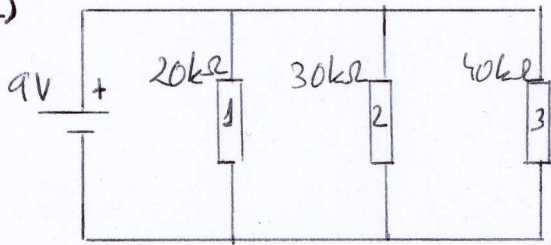


29. a)



R_1, R_2 and R_3 are three resistors in a parallel connection:

Equivalent resistance:

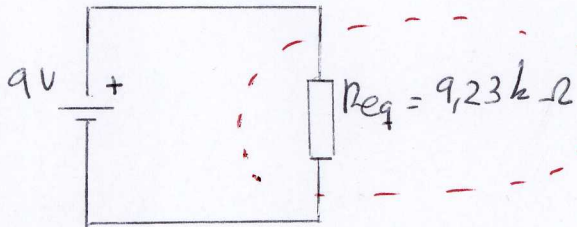
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{40} = \frac{6+4+3}{120}$$

$$\frac{1}{R_{eq}} = \frac{13}{120} \rightarrow R_{eq} = \frac{120}{13}$$

$$R_{eq} = 9,23 \text{ k}\Omega$$

b)



In order to calculate the total current we apply Ohm's law to the resistor

↳ Voltage across the resistor is 9V because it's the only resistor in this circuit, so V is the same as the value of voltage across the battery

$$V = IR \rightarrow I = \frac{V}{R} = \frac{9V}{9,23 \text{ k}\Omega}$$

$$I = 0,975 \text{ mA} *$$

* This is a very low value. The reason for that is that the resistance values in the resistors is very high.

c) 1. Voltage across each resistor in the original circuit is the same (because they are in a parallel connection)

2. Electrons only go through one resistor on a circle on the circuit so $V_1 = V_2 = V_3 = 9V$

In order to calculate current through each resistor we must apply Ohm's law:

$$I = \frac{V}{R} \rightarrow \begin{cases} I_1 = \frac{V}{R_1} = \frac{9}{20} = 0,45 \text{ mA} \\ I_2 = \frac{V}{R_2} = \frac{9}{30} = 0,30 \text{ mA} \\ I_3 = \frac{V}{R_3} = \frac{9}{40} = 0,225 \text{ mA} \end{cases}$$

As you can see:

$$I_{TOT} = I_1 + I_2 + I_3$$

$$0,975 = 0,45 + 0,30 + 0,225 = 0,975$$

d) As we have said in section c) $V = 9V$

$$P = I \cdot V = 0,975 \text{ mA} \cdot 9V = 8,775 \text{ mW}$$

e) Battery
Resistors

$$P_1 = I_1 \cdot V = 0,45 \cdot 9 = 4,05 \text{ mW}$$

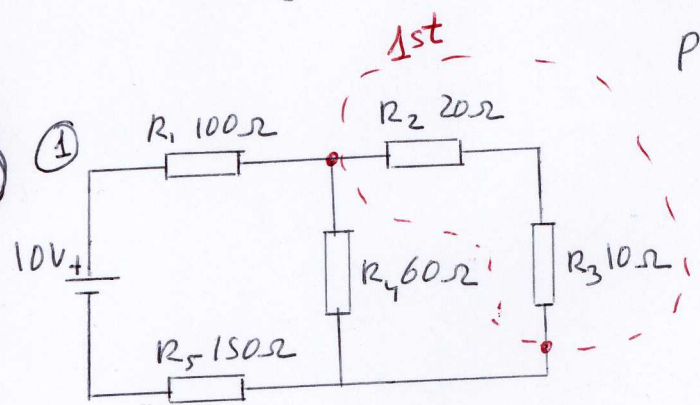
$$P_2 = I_2 \cdot V = 0,3 \cdot 9 = 2,7 \text{ mW}$$

$$P_3 = I_3 \cdot V = 0,225 \cdot 9 = 2,025 \text{ mW}$$

As you can check $P = P_1 + P_2 + P_3$

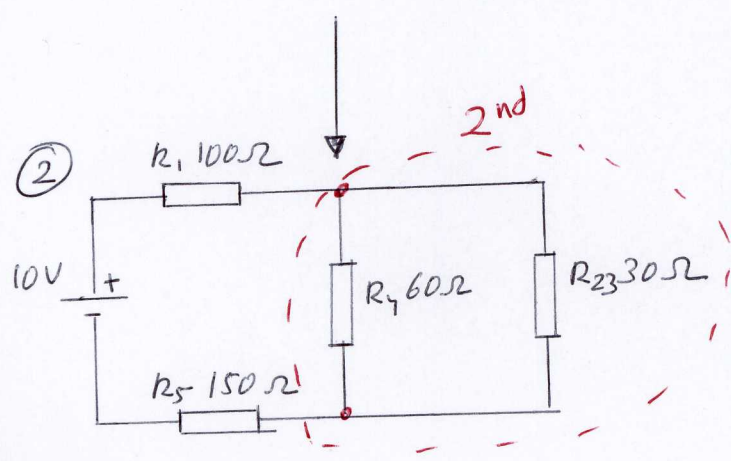
$$P = 4,05 + 2,7 + 2,025 = 8,775 \text{ mW}$$

30.



We start simplifying R_2 and R_3 because they are in a series connection

$$R_{23} = R_2 + R_3 = 20 + 10 = 30\Omega$$

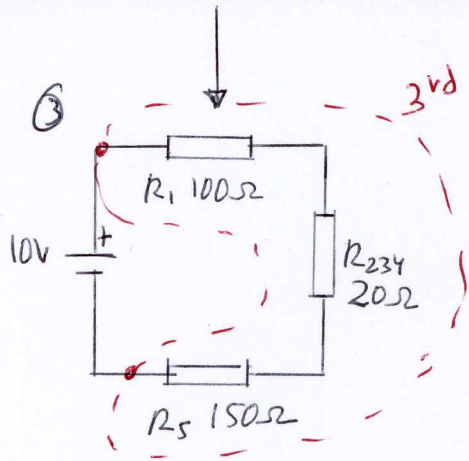


Now we simplify R_4 and R_{23} . they are in a parallel connection

$$\frac{1}{R_{234}} = \frac{1}{R_4} + \frac{1}{R_{23}} = \frac{1}{60} + \frac{1}{30}$$

$$\frac{1}{R_{234}} = \frac{1+2}{60} = \frac{3}{60} \rightarrow R_{234} = \frac{60}{3}$$

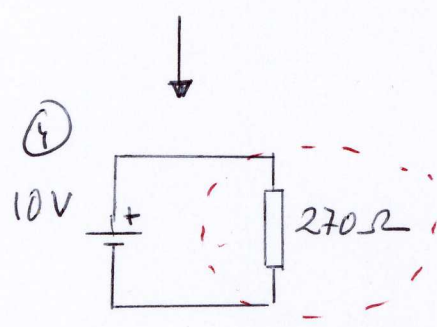
$$R_{234} = 20\Omega$$



Finally we simplify this three resistors (R_1, R_{234}, R_5) in series connection

$$R_{eq} = R_1 + R_{234} + R_5$$

$$R_{eq} = 100 + 20 + 150 = \boxed{270\Omega}$$



We are applying Ohm's law to calculate current through the resistor

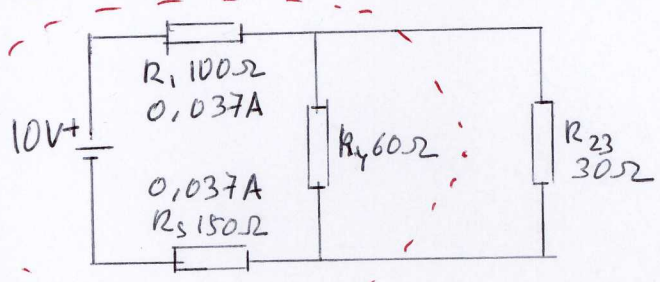
$$V = IR \rightarrow I = \frac{V}{R} = \frac{10}{270} = 0,037 A$$

Now that we have solved the equivalent circuit, we need to move backwards in order to find the other current values.

1. Current through the 10V battery is the same as the current through the equivalent resistor because they are in series connection
2. If you take a look to circuit (3) you will see that current in the battery has to be the same as current through R_1, R_{234} and R_5 because they are in a series connection

$$\boxed{I_1 = I_{234} = I_5 = 0,037 A}$$

3. Now we are calculating I_4 . Take a look to circuit (2)



In order to calculate I_4 using Ohm's law we need to calculate V_4 first

$$V_{battery} = V_1 + V_4 + V_5 \rightarrow$$

$$V_4 = V_{battery} - V_1 - V_5$$

We already know R_1 and $I_1 \rightarrow V_1 = I_1 R_1 = 0,037 \cdot 100 = 3,7V$
 R_5 and $I_5 \rightarrow V_5 = I_5 R_5 = 0,037 \cdot 150 = 5,55V$

So:

$$V_4 = 10 - 3,7 - 5,55 = 0,75V$$

And

$$I_4 = \frac{V_4}{R_4} = \frac{0,75}{60} = 0,0125A$$

$$I_4 = 0,0125A$$

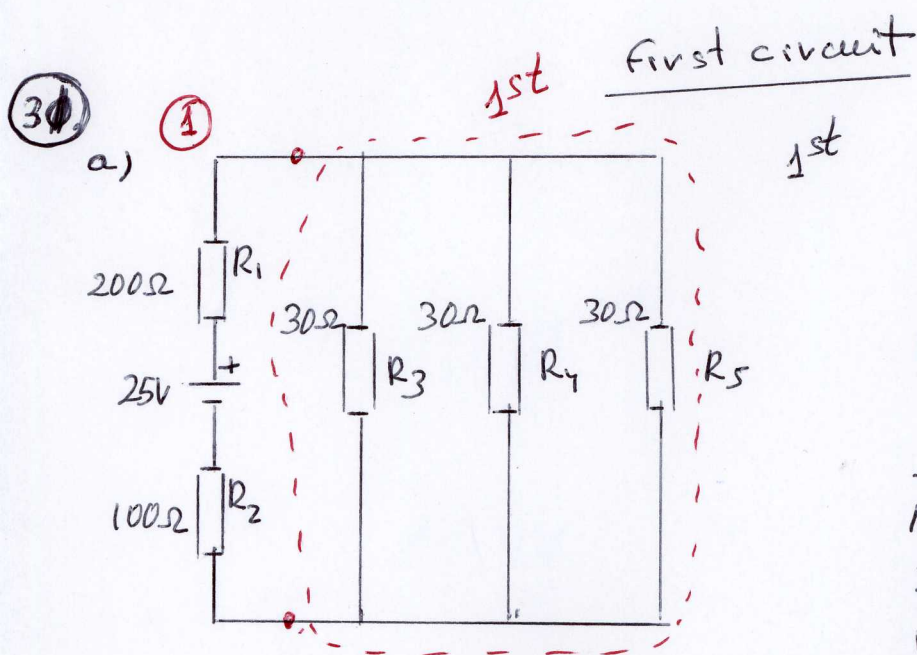
4. This is very simple.

$$I_1 = I_4 + I_{23} \rightarrow I_{23} = I_1 - I_4$$

$$I_{23} = 0,037 - 0,0125 = 0,0245A$$

$$I_{23} = I_2 = I_3 = 0,0245A$$

↑ because series connection.



1st R_3, R_4 and R_5 is a parallel connection, so:

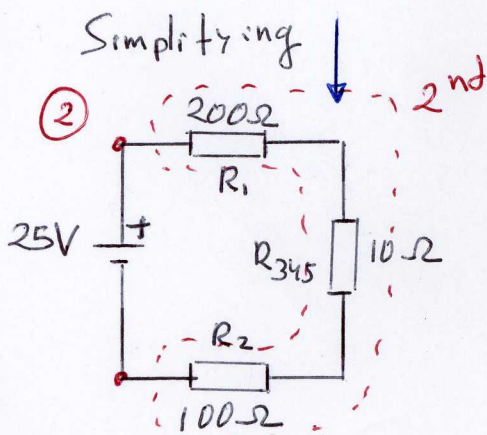
$$\frac{1}{R_{345}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

$$\frac{1}{R_{345}} = \frac{1}{30} + \frac{1}{30} + \frac{1}{30} = \frac{3}{30}$$

Inverting the outcome:

$$\frac{1}{R_{345}} \rightarrow \frac{R_{345}}{1} = \frac{30}{3} \rightarrow R_{345} = 10\Omega$$

Simplifying



2nd

R_1, R_{345} and R_2 is a series connection, so:

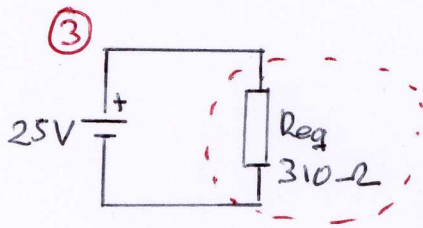
$$R_{eq} = R_1 + R_{345} + R_2$$

$$R_{eq} = 200 + 10 + 100 = 310\Omega$$

$$R_{eq} = 310\Omega$$

b)

Simplifying



By being all these circuits equivalent
current ~~across~~ ^{through} this last equivalent
resistor is the same as current in all
the other batteries.

$$V = IR$$

$$I = \frac{V}{R} \rightarrow I = \frac{25}{310} = 0,081 \text{ A}$$

$$I = 81 \text{ mA}$$

c)

We need to find current ~~across~~ ^{through} all five resistor but

$I_1 = I_2 = I_{\text{battery}}$ by being in a series connection

$$I_1 = I_2 = 81 \text{ mA}$$

It would be possible to calculate I_3, I_4 and I_5 following
the same technique we have described in exercise ③0.
but if you stop and think for a moment you can see
that R_3, R_4 and R_5 are in a parallel connection and
their resistance values are just the same. That means
that the total current it is going to divide in three
equal parts

$$I_3 = I_4 = I_5 = \frac{I_{\text{battery}}}{3} = \frac{81}{3} = 27 \text{ mA}$$

$$I_3 = I_4 = I_5 = 27 \text{ mA}$$

d) and e)

knowing all R and I values this section is very
simple:

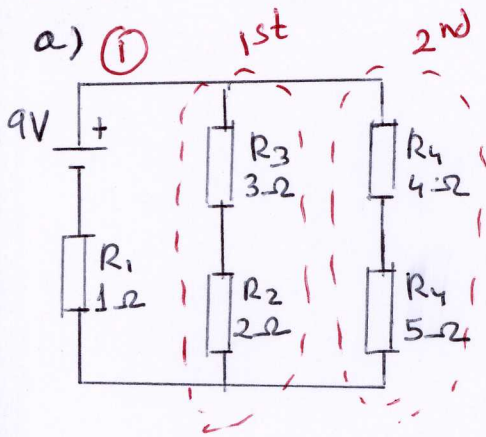
$$V = I \cdot R$$

$$P = I \cdot V$$

We just have to complete the table

	$R (\Omega)$	$I (\text{mA})$	$V (\text{V})$	$P (\text{W})$
1	200	81	16,2	1,312
2	100	81	8,1	0,656
3	30	27	0,81	0,022
4	30	27	0,81	0,022
5	30	27	0,81	0,022

} They are
identical.

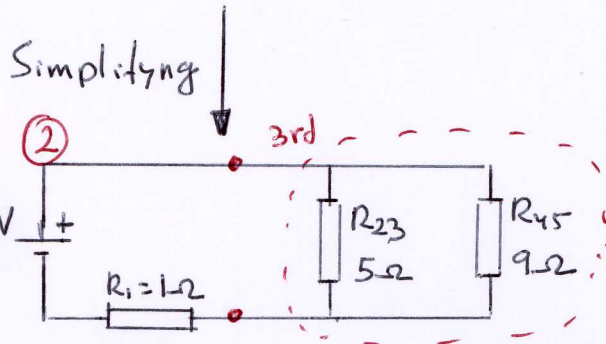


1st R_2 and R_3 is a series connection

$$R_{23} = R_2 + R_3 = 2 + 3 = 5 \Omega$$

2nd R_4 and R_5 is again a series connection.

$$R_{45} = R_4 + R_5 = 4 + 5 = 9 \Omega$$

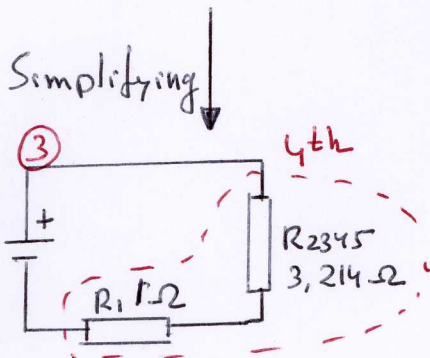


3rd R_{23} and R_{45} is a parallel connection

$$\frac{1}{R_{2345}} = \frac{1}{R_{23}} + \frac{1}{R_{45}} = \frac{1}{5} + \frac{1}{9} = \frac{14}{45}$$

Inverting:

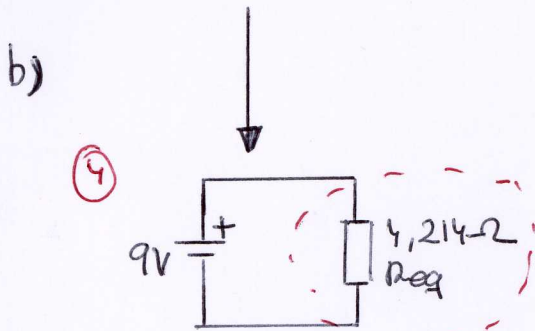
$$\frac{R_{2345}}{1} = \frac{45}{14} \quad R_{2345} = 3,214 \Omega$$



4th this is a series connection:

$$R_{eq} = R_1 + R_{2345} = 1 + 3,214$$

$$R_{eq} = 4,214 \Omega$$



By being all 4 circuit equivalents current across R_{eq} is the same as current ~~across~~ ^{through} the batteries in all four systems

$$V = I \cdot R \rightarrow I = \frac{V}{R} = \frac{9}{4,214} \rightarrow I = 2,14 A$$

c) We have to calculate current ~~across~~ ^{through} all five resistors. I_1 is the same as $I_{battery}$ as they are in series

$$I_1 = I_{battery} = 2,14 A = I_1$$

I_2 and I_3 are going to be the same because they are in a series connection

Applying Ohms law to R_{23} in circuit ②

$$V_{23} = I_{23} \cdot R_{23} \rightarrow I_{23} = \frac{V_{23}}{R_{23}}$$

But we have a problem. We need to calculate first V_{23} .

Take a look to circuit number ② & ③

③ R_1 and R_{2345} are in a series connection so:

$$V_{\text{battery}} = V_1 + V_{2345}$$

② R_{23} and R_{45} are in a parallel connection so:

$$V_{23} = V_{45} = V_{2345}$$

That means that by calculating V_1 we can obtain

V_{23} and also V_{45}

$$\textcircled{V_1} \rightarrow V_1 = I_1 R_1 = 2,14 \cdot 1 = 2,14 \text{ V}$$

$$\rightarrow V_{2345} = V_{\text{battery}} - V_1 = 9 - 2,14 = 6,86 \text{ V}$$

$$\text{So: } I_{23} = \frac{V_{2345}}{R_{23}} = \frac{6,86}{5} = 1,372 \text{ A}$$

$$\boxed{I_2 = I_3 = 1,372 \text{ A}}$$

We can now calculate I_4 and I_5 in the same way

$$I_{45} = \frac{V_{2345}}{R_{45}} = \frac{6,86}{9} = 0,762 \text{ A}$$

$$\boxed{I_4 = I_5 = 0,762 \text{ A}} *$$

* If you take a moment to understand the problem it would be also possible to do:

$$I_4 = I_{\text{battery}} - I_2$$

d) and e)

Just like we did in section part 1 with the first circuit we just to complete the table

	R (Ω)	I (A)	V (V)	P (W)
1	1	2,14	2,14	4,58
2	2	1,372	2,744	3,76
3	3	1,372	4,116	
4	4	0,762	3,048	2,32
5	5	0,762	3,81	

$V = I \cdot R$

32.

This exercise is the same as exercise 30. The only difference is that there are two batteries in a series connection.

We need to simplify those batteries

$$V_{eq} = V_{battery 1} + V_{battery 2}^*$$

$$V_{eq} = 11 + 9 = 20V$$

You can now solve the system following the same technique as the one described on exercise 30.

* We are adding voltages because the negative pole of one of the batteries is connected to the positive pole on the other battery.